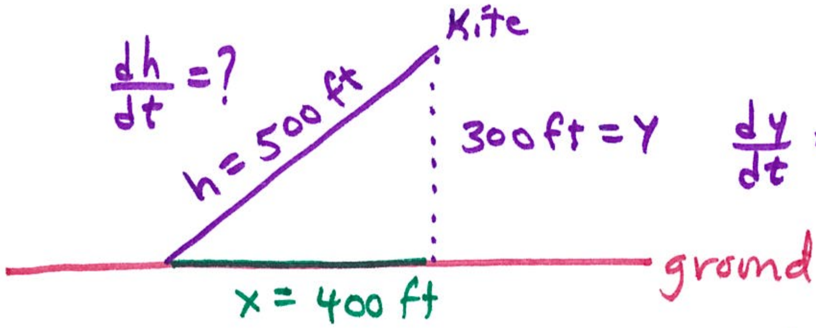
Section 5.6 pg. 256

14.

$\frac{dh}{dt} = ?$



$\frac{dy}{dt} = 0$

$\frac{dx}{dt} = 25$ ft/sec

Find $\frac{dh}{dt}$

$x^2 + y^2 = h^2$

$\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y \cdot \frac{dy}{dt} = \cancel{2}h \cdot \frac{dh}{dt}$

$400(25) + 300(0) = 500 \frac{dh}{dt}$

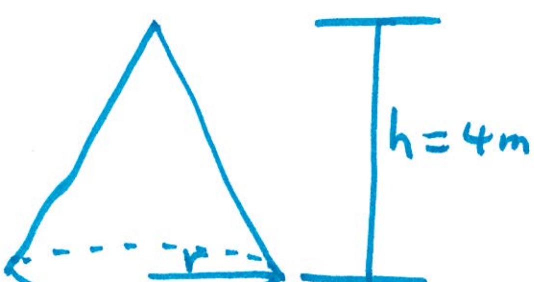
$400(25) = 500 \frac{dh}{dt}$

$\frac{4\cancel{00}(25)}{5\cancel{00}} = \frac{dh}{dt}$

$20 = \frac{dh}{dt}$

Inge must let out the string at a rate of 20 ft/sec

16.



$\frac{dV}{dt} = 10 \ m^3/min$

$V = \frac{1}{3}\pi r^2 h$

$V(r) = \frac{1}{3}\pi r^2 \left(\frac{3}{4}r\right)$

$h = \frac{3}{8}d \qquad h = 4m$

$h = \frac{3}{8}(2r)$

$h = \frac{3}{4}r \qquad 4 = \frac{3}{4}r$

$\frac{16}{3} = r$

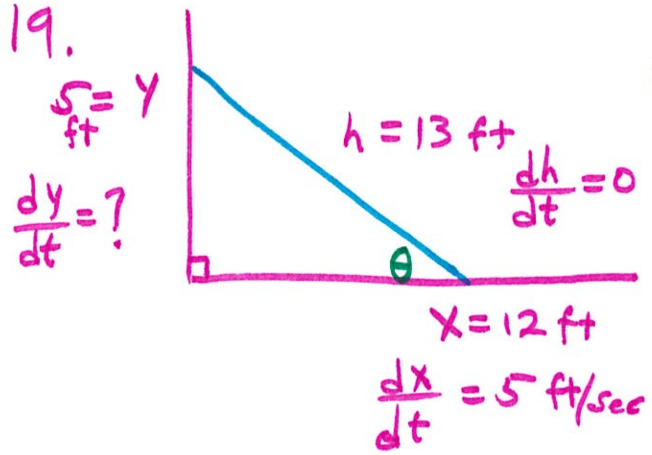
a. and b. $V(r) = \frac{1}{4}\pi r^3 \qquad \frac{dV}{dt} = \frac{3}{4}\pi r^2 \frac{dr}{dh} \qquad 10 = \frac{3}{4}\pi\left(\frac{16}{3}\right)^2 \frac{dr}{dt}$

$10 = \frac{64}{3}\pi \frac{dr}{dt} \qquad \frac{dr}{dt} = \frac{30}{64\pi}$ m/min $\qquad \frac{dr}{dt} = \frac{15}{32\pi}$ m/min ≈ 0.149 m/min

≈ 14.9 cm/min

$h = \frac{3}{4}r \qquad \frac{dh}{dt} = \frac{3}{4}\frac{dr}{dt} \qquad \frac{dh}{dt} \approx \frac{3}{4}(14.9) \approx 11.18$ cm/min

19.



Ⓐ

$$x^2 + y^2 = h^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2h\frac{dh}{dt}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = h\frac{dh}{dt}$$

$$12(5) + 5\cdot\frac{dy}{dt} = 13(0)$$

$$\frac{dy}{dt} = \frac{-12(5)}{5}$$

$$\frac{dy}{dt} = -12 \text{ ft/sec}$$

The ladder is sliding down the wall at a rate of 12 ft/sec

Ⓑ $A = \frac{1}{2}xy$ $\qquad \frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\frac{dy}{dt}$

$$\frac{dA}{dt} = \frac{1}{2}(5)(5) + \frac{1}{2}(12)(-12)$$

$$= 12\tfrac{1}{2} - 72$$

$$= 59\tfrac{1}{2} \text{ ft}^2/\text{sec}$$

The area is changing (increasing) at a rate of $59\frac{1}{2}$ ft²/sec.

Ⓒ $\tan\theta = \frac{y}{x}$ $\qquad$ Find $\frac{d\theta}{dt}$ $\qquad \theta = \tan^{-1}\left(\frac{5}{12}\right)$

$\theta \approx 0.3948$ radians

$$\sec^2\theta\cdot\frac{d\theta}{dt} = \frac{\frac{dy}{dt}\cdot x - y\cdot\frac{dx}{dt}}{x^2}$$

$$\left(\frac{13}{12}\right)^2\frac{d\theta}{dt} = \frac{(-12)(12) - (5)(5)}{144}$$

$$\frac{d\theta}{dt} = \frac{-169}{144}\cdot\frac{144}{169}$$

$$\frac{d\theta}{dt} = -1 \text{ radian/sec.}$$

27. Surface area of a sphere is $A = 4\pi r^2$

$d = 20\text{ cm} \quad r = 10\text{ cm} \quad \frac{dV}{dt} = -8\text{ mL/min} = -8\text{ cm}^3/\text{min}$

$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$-8 = 4\pi(10)^2 \frac{dr}{dt}$

$\frac{-8}{400\pi} = \frac{dr}{dt}$

$\frac{-1}{50\pi} = \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{-1}{50\pi}\text{ cm/min}$

$A = 4\pi r^2 \quad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 8\pi(10)\left(\frac{-1}{50\pi}\right)$

$\frac{dA}{dt} = -\frac{8}{5}\text{ cm}^2/\text{min}$

$\frac{dA}{dt} = -1.6\text{ cm}^2/\text{min}$

The surface Area is decreasing at the rate $1.6\text{ cm}^2/\text{min}$